Perms and Combs Practice QUIZ II SOLUTIONS

1. In a particular jurisdiction, license plates consist of any three letters, the first of which cannot be I or O, followed by any three non-repeating digits. Determine the number of possible license plates.

2 $24 \times 26 \times 26 \times 10 \times 9 \times 8 = 11\ 681\ 280$

Not I or O   No Repeating

2. Consider the letters in the word "REGRETTABLE". Determine the number of possible arrangements if:

(a) There are no restrictions

Divide out identicals

$$\frac{11!}{3!\,2!\,2!} = 1\ 663\ 200$$

E's   T's   R's

(b) Each arrangement must start with an B

4 Arrange "REGRETALE" $\dfrac{10!}{3!\,2!\,2!} = 151\ 200$

(c) Each arrangement must start with an E

Arrange "RGRETABLE" $\dfrac{10!}{2!\,2!\,2!} = 45\ 360$

(d) Each arrangement must start and end with a T

Arrange "REGREABLE"

$$\frac{9!}{3!\,2!} = 30\ 240$$

Only 2 E's Now

3. In a particular family of 8 children, there are 5 boys and 3 girls. A photographer is hired to take a series of family pictures of the children only, where they're arranged in a row. How many ways can this be done if:

a) There are no restrictions $8! = 40\ 320$

b) If the boys must be on the left side, and the girls on the right side, girls on the right:

5 B B B B B G G G $5! \times 3! = 720$

c) If the girls must all be together

Count Gs as one   Arrange Gs

Count Girls as ONE → $\boxed{B}\boxed{B}\boxed{B}\boxed{B}\boxed{B}\boxed{GGG} = 6! \times 3!$

$= 4320$

d) If two of the girls, Elizabeth and Katie, **cannot** be together

TOTAL − Arrangements where E & K ARE together

$= 8! - 7!\,2! \Rightarrow = 30\ 240$

;]/. Count E & K as ONE   arrangements of E & K

e) If the photograph consists of just three children – 2 boys and 1 girl. (The photographer selects the three, then arranges them in a row)

$$\underbrace{{}_5C_2 \times {}_3C_1}_{\text{select}} \times \underbrace{3!}_{\text{then arrange}} = 180$$

4. **NR** The three ➔ problems that can be solved using $\binom{n}{r}$ are 2, 3, 4, and

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1	The number of different arrangements using all the letters in the word MATHY
2	The number of unique 5 player teams that can be selected from 8 boys and 9 girls
3	The number of line segments that can be drawn using the vertices of an 8-sided polygon that are marked on a circle.
4	The number of different way to choose 4 specialty donuts from a display of 6 different donuts at a coffee shop
5	The number of different ways to assign the job of chairperson, vice-chair, and secretary for a committee from 6 people.

5. Algebraically determine the

$$\frac{n!}{(n-2)!\,2!}$$

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$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = 21 \qquad n=7 \text{ or } \cancel{-6}$$

$$n(n-1) = 21 \times 2$$

5. Consider the letters in the word SMILE and FROG. The consonants are S, M, L, F, R, and G.

(a) How many ways can any 2 letters be selected from the word SMILE? *(That is, how many two-letter groups, not arrangements, are possible?)*

$${}_5C_2 = 10$$

(b) How many ways can any 2 letters be selected from the word SMILE and any 2 letters be selected from the word FROG?

$${}_5C_2 \times {}_4C_2 = 60$$

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(c) How many ways can the letters in any four-letter word be arranged? *(Assuming all letters are different)*

$$4! = 24$$

(d) How many different 4-letter arrangements are possible using any 2 letters from the word SMILE and any 2 letters from the word FROG?

$$\underbrace{{}_5C_2 \times {}_4C_2}_{\text{select}} \times \underbrace{4!}_{\text{...then arrange}} = 1440$$

6. A student council consists of 7 girls and 5 boys. A subcommittee of four council members is needed to coordinate a school dance. How many ways can this be done if:

(a) There are no restrictions

$${}_{12}C_4 = 495$$

(b) There must be exactly 2 boys and 2 girls

$${}_5C_2 \times {}_7C_2 = 210$$

(c) There must be exactly 2 boys and 2 girls, and the council president Claire (girl) must be on the subcommittee? New question: Select 2 boys, 1 Girl

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$${}_5C_2 \times {}_6C_1 = 60$$

(d) If either Claire **or** the vice-president David (boy) must on the subcommittee. (Hint: Consider two different cases)

Claire / David

Ans from part c $\rightarrow$

$$60 + {}_4C_1 \times {}_7C_2 = 144$$

(e) There must be at least one boy on the subcommittee.

1 Boy or 2 Boys or 3 Boys or 4 Boys

$${}_5C_1 \times {}_7C_3 + {}_5C_2 \times {}_7C_2 + {}_5C_3 \times {}_7C_1 + {}_5C_4$$

$$= 460$$

Short way: ${}_{12}C_4 - {}_7C_4$ $\leftarrow$ All Girls (No Boys)

BONUS: A pizzeria offers a \$9.99 special, where a medium pizza with up to five toppings can be ordered. (That is, a customer can order less toppings if they wish) Assuming double-toppings are not permitted, a plain cheese pizza counts as no toppings, and there are 12 toppings to choose from, determine the total number of orders possible.

$${}_{12}C_0 + {}_{12}C_1 + {}_{12}C_2 + {}_{12}C_3 + {}_{12}C_4 + {}_{12}C_5$$

$$= 1586$$

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