SOLUTIONS Math 30-1 Transformations Practice Exam

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1. Reflection about line $x = 0$ is **horizontal**, so replace $x \to -x$. Then, for horizontal translation, replace $x \to x + 3$, giving us $y = f[-(x+3)]$ which simplifies to $y = f(-x-3)$.

ANSWER: **D**

NR #1 For $g(x)$, the transformation is to take all points on $f(x)$ and
- Horizontally shift 2 left *So subract 2 from all x-coords*
- Vertically shift k up *So add "k" to all y-coords*

The DOMAIN of $f(x) = \sqrt{\underbrace{x+3}} - 1$ is...

Can't sq root negatives $x + 3 \geq 0$

$x \geq -3$ $\xrightarrow{\text{shift 2 left}}$ $\boldsymbol{x \geq -5}$

The RANGE of $f(x) = \sqrt{x+3} \underbrace{-\,1}$ is...

Basic graph of $y = \sqrt{x}$ shifted 1 down $y \geq -1$ $\xrightarrow{\text{shift "k" up}}$ $y \geq -1 + k$

Simplifies to: $\boldsymbol{y \geq k - 1}$

ANSWER: **94**

NR #2 Apply vert. str. factor of 3 to $y = 2x^2 + 1$, to get

$y = \mathbf{3}(2x^2 + 1)$

$\boldsymbol{y = 6x^2 + 3}$

Then, apply vert. shift up 2 to get $y = 6x^2 + 3 + \mathbf{2}$

$\boldsymbol{y = 6x^2 + 5}$

ANSWER: **65**

NR #3 *Jagmeet sketches the graph of $h(x) = 4f(-x) - 3$ ....*

All pts $(x, y) \to (-x, 4y - 3)$

$(-4, 2) \to (\mathbf{4}, \mathbf{5})$ $\quad 4(2) - 3$

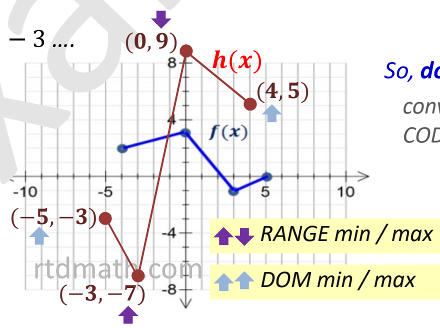
First (farthest left) pt on graph of $f(x)$ $\quad -(-4)$

$(0, 3) \to (\mathbf{0}, \mathbf{9})$

$(3, -1) \to (\mathbf{-3}, \mathbf{-7})$

$(5, 0) \to (\mathbf{-5}, \mathbf{-3})$

Plot each of these points!



*So, **domain** is* $[-5, 4]$ and ***range*** is $[-7, 9]$

convert to CODES 3 4 2 9

ANSWER: **3429**

2. Transformations for $g(x)$ are a vertical reflection *(replace $y \to -y$)* a vertical stretch, and a vertical shift one unit up.

So all pts $(x, y) \to (x, -2y + 1)$

Now, the RANGE of $f(x)$ *(shown)* is $[-4, \infty)$

Which on $g(x)$ becomes $(-\infty, \mathbf{9}]$

$(-2)(-\mathbf{4}) + 1$

ANSWER: **A**

3. Transformations for $h(x)$ are a horizontal stretch, factor of 2, and a horiz. translation 4 units right.

**remember to first FACTOR:* $h(x) = f[\frac{1}{2}(x - 4)]$

Now, the DOMAIN of $f(x)$ *(shown)* is $[-5, \infty)$

Which on $g(x)$ becomes $[\mathbf{6}, \infty)$

$(2)(-\mathbf{5}) + 4$

ANSWER: **C**

4. The largest x-intercept on $h(x)$ *is* -1. After a horiz. str. factor of 2 and shift 4 right, that becomes

$\boldsymbol{x = 2}$

$(2)(-\mathbf{1}) + 4$

ANSWER: **D**

5. On $f(x)$ the x-coord of P is 3

Horiz. str by a factor of 3 changes this to **9** $\quad (3)(\mathbf{3})$

Then horiz. shift 9 units right changes this to $\boldsymbol{m = 18}$ $\quad \mathbf{9} + 9$

ANSWER: **D**

6. An equation for $k(x)$ can be found by replacing, in the equation for $f(x)$

$\boldsymbol{x}$ with $\boldsymbol{\frac{1}{3}x}$ *then, in the resulting expression* $\frac{1}{3}x$ *.....* Replacing x with $\boldsymbol{x - 9}$

Horiz str. factor of 3

$\boldsymbol{\frac{1}{3}x \to \frac{1}{3}(x - 9)}$

Horiz shift 9 right

So net effect: $\boldsymbol{k(x) = f\left[\frac{1}{3}(x - 9)\right]}$

ANSWER: **B**

7. The domain of $f(x)$ is $x \geq 9$

***Think:** Basic graph of $y = \sqrt{x}$ (domain $x \geq 0$) shifted 9 units right*

Then we apply the mapping rule

$3(\mathbf{9}) - 6$ *to get* $\boldsymbol{x \geq 21}$

ANSWER: **C**

8. The total width of $f(x)$ is **6** units. *($x = 2$ to $x = 8$)*

The total width of $g(x)$ is **1.5** units. *($x = 0.5$ to $x = 2$)*

➔Therfore $g(x)$ is 1/4th as wide $(1.5 \div 6)$

HORIZ STR. FACTOR OF 1/4 $\quad \boldsymbol{b}$ *value in equation is 4 (reciprocal)*

The total height of $f(x)$ is **3** units. *($y = 0$ to $y = 3$)*

The total height of $g(x)$ is **9** units. *($y = 2$ to $y = 11$)*

➔Therfore $g(x)$ is 3 times "taller" **VERT STR. FACTOR OF 3**

BUT there is also a vertical shift, k.

***Think:** $g(x)$ "should" have invariant points on the x-axis, from the vertical stretch. (at the MIN)*

However the MIN y-value of $g(x)$ is 2

➔Therfore k is 2 units up

ANSWER: **A**

9. Reflection about line $x = 0$ replace $x \to -x$. **AND**
Reflection about line $y = 0$ / think of one of two ways. Either replace $y \to -x$ or *make the entire expression negative.*

$$g(x) = -[2(-x)^2 - 3(-x) + 5]$$

For **Vert.** Refl. For Horiz. Refl. (*— sign* ***inside***)
(include square brackets around all terms)

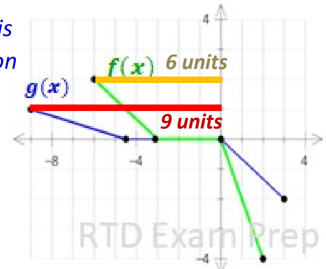
$$g(x) = -[2x^2 + 3x + 5]$$

$$\boldsymbol{g(x) = -2x^2 - 3x - 5}$$

ANSWER: **A**

NR #5 Many options! Use the "furthest left" point on each graph.

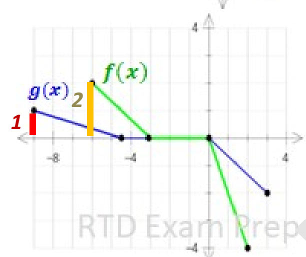
On $f(x)$ the furthest left point is ***6 horiz. units*** *from the y-axis, on $g(x)$ it's* ***9 horiz. units.***



➔ Horiz. str, factor of $9/6$

reduces to $\frac{3}{2}$

On $f(x)$ the highest point is ***2 vert. units*** *from the x-axis, on $g(x)$ it's* ***1 unit.***



➔ vert. str, factor of $\frac{1}{2}$

SO… $a = \frac{1}{2} \begin{smallmatrix}\leftarrow m \\ \leftarrow n\end{smallmatrix}$
(vert. str.)

AND… $b = \frac{2}{3} \begin{smallmatrix}\leftarrow p \\ \leftarrow q\end{smallmatrix}$
(reciprocal of horiz. str)

ANSWER: **1223**

NR #4 Start with $f(x)$ …. *DOMAIN* is $[-2, \infty)$ and *RANGE* is $[-4, \infty)$
"left to right" "bottom to top"

❖ *$g(x)$ involves a Vert. Str, factor of 3/2, a horiz. reflection, and a shift 2 right and 2 up.*

All pts $(x, y) \to (\underbrace{-x - 2}_{\text{affects domain}}, \underbrace{\tfrac{3}{2}y + 2}_{\text{affects range}})$

DOMAIN of $g(x)$ …. $-(-\mathbf{2}) - \mathbf{2}$ ➡ $\boldsymbol{x \le 0}$ **REF. # 1**
(reverse direction / graph points LEFT now)

RANGE of $g(x)$ …. $\frac{3}{2}(-\mathbf{4}) + \mathbf{2}$ ➡ $\boldsymbol{y \ge -4}$ **REF. #7**

❖ *$h(x)$ is the INVERSE, $(x, y) \to (y, x)$ Domain and range* ***switch***

DOMAIN of $h(x)$ ➡ $\boldsymbol{x \ge -4}$ **REF. # 5**
the RANGE of $f(x)$

RANGE of $h(x)$ ➡ $\boldsymbol{y \ge -2}$ **REF. # 8**
the DOMAIN of $f(x)$

ANSWER: **1758**

10. First apply vertical reflection

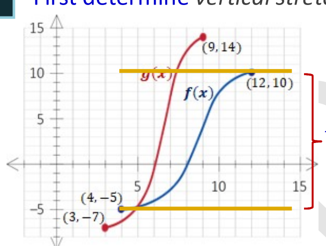
$= -[2x^2 + 3x - 5]$ *— in front of entire expression*
$= -2x^2 - 3x + 5$

Then apply horiz shift 1 left *Replace $\boldsymbol{x}$ with "$\boldsymbol{x + 1}$"*

$= -2(x + 1)^2 - 3(x + 1) + 5$
$= -2(x^2 + 2x + 1) - 3x - 3 + 5$
$= -2x^2 - 4x - 2 - 3x + 2$
$= -2x^2 - 7x$

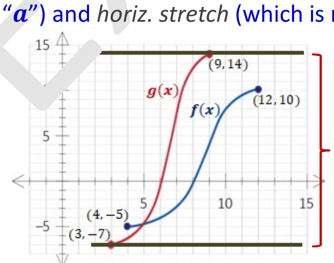
ANSWER: **B**

NR #6 First determine *vertical stretch* (which is "$\boldsymbol{a}$") and *horiz. stretch* (which is reciprocal of "$\boldsymbol{b}$")



$f(x)$ is 15 units "tall"

$f(x)$ is 8 units "wide"
(left to right … $x = 4$ to $x = 12$)



and $g(x)$ is 21 units "tall"

and $g(x)$ is 6 units "wide"
(left to right … $x = 3$ to $x = 9$)

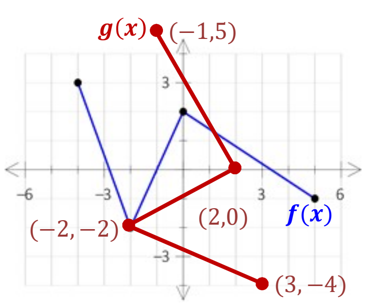
➔ Vert. str, factor of 21/15

Decimal form is $\mathbf{1.4}$

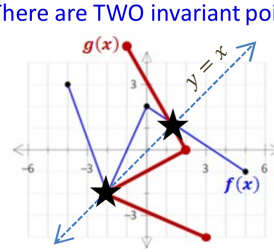
ANSWER: **1408**

➔ Horiz. str, factor of 6/8 *Decimal form rounds to* $\mathbf{0.8}$

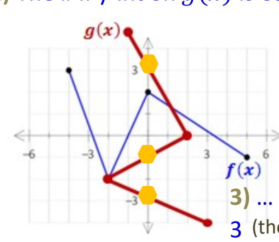
NR #7 The graph of $g(x)$ can be obtained by switching all coordinates $(x, y) \to (\boldsymbol{y, x})$



1) There are TWO invariant points



2) The # if y-int on $g(x)$ is equal to the # of x-ints on $f(x)$ ➔**3**



3) … and the largest x-coord is 3 (the largest y-coord on $f(x)$)

ANSWER: **233**

11. Switch x and y, then isolate y

first re-write $f(x)$ in terms of y

$y = \sqrt{x+4} - 1$

square both sides

$x = \sqrt{y+4} - 1$ ➡ $x + 1 = \sqrt{y+4}$ ➡ $(x+1)^2 = (\sqrt{x+4})^2$

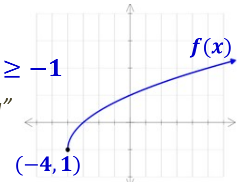
➡ $(x+1)^2 = y + 4$ ➡ $\mathbf{y = (x+1)^2 - 4}$ ANSWER: **B**

12. The domain of $g(x)$ is the range of $f(x)$

Range of $f(x)$ is $y \geq -1$

So domain of $g(x)$ is $\mathbf{x \geq -1}$

Graph is a "half-parabola"



And the y-int of $g(x)$ is $\mathbf{y = -3}$

(The x-int of $f(x)$)

ANSWER: **A**

WR Question 1

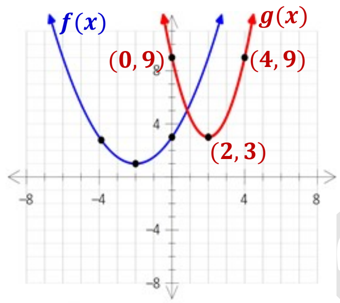
(a) Vert. str. by factor of 3, **then** vert. translation 4 units right

$(x, y) \rightarrow (x + 4, 3y)$

$(-4, 3) \rightarrow (0, 9)$

$(-2, 1) \rightarrow (2, 3)$

$(0, 3) \rightarrow (4, 9)$



(b) *Equation of $f(x)$....* $y = a(x+\mathbf{2})^2 + \mathbf{1}$ ➡ $(\mathbf{3}) = a((\mathbf{0}) + 2)^2 + 1$

➡ $2 = a(2)^2$

➡ $a = \frac{2}{4}$ ➡ $\mathbf{a = \frac{1}{2}}$

Use any other pt on the graph to solve for $\mathbf{a}$ ***(Such as $(0, 3)$!)***

So, equation of $f(x)$ is .... $\mathbf{f(x) = \frac{1}{2}(x+2)^2 + 1}$

For equation of $g(x)$ we could use a similar method (sub in vertex then solve for "$\mathbf{a}$")

OR we could simply apply the transformations to the equation of $f(x)$....

$g(x) = \mathbf{3}\left[\frac{1}{2}((\mathbf{x-4}) + 2)^2 + 1\right]$

horiz. translation 4 right

Vert. str, factor of 3

So, equation of $g(x)$ is .... $\mathbf{g(x) = \frac{3}{2}(x-2)^2 + 3}$

(c) Here we saw that horizontally tranlating the vertex 4 right achieved the same result as horizontally reflecting the graph of $y = f(x)$.

This can be verified by applying the horiz. reflection to the equation of $y = f(x)$ ....

$y = \frac{1}{2}((\mathbf{-x}) + 2)^2 + 1$ ➡ $y = \frac{1}{2}(-x+2)^2 + 1$ ➡ $y = \frac{1}{2}[-1(x-2)]^2 + 1$

Then factor out a "$\mathbf{-1}$" from inside the brackets

Replace "$\mathbf{x}$" with "$\mathbf{-x}$" for horiz. reflection

➡ $y = \frac{1}{2}(-1)^2(x-2)]^2 + 1$ ➡ $y = \frac{1}{2}(x-2)]^2 + 1$

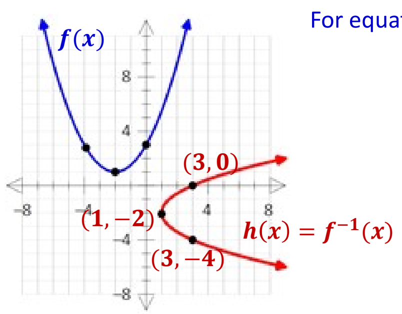
Same resulting equation as replacing "$\mathbf{x}$" with "$\mathbf{x-4}$" !

(c) For graph of $y = h(x)$, (inverse) switch all points $(x, y) \rightarrow (y, x)$

$(-4, 3) \rightarrow (3, -4)$

$(-2, 1) \rightarrow (1, -2)$

$(0, 3) \rightarrow (3, 0)$



For equation of $y = h(x)$, switch $\mathbf{x}$ and $\mathbf{y}$ in the equation and isolate y.

$y = \frac{1}{2}(x+2)^2 + 1$

$\mathbf{x} = \frac{1}{2}(\mathbf{y}+2)^2 + 1$

$\mathbf{x - 1} = \frac{1}{2}(y+2)^2$

$2(x-1) = (y+2)^2$

sq. root both sides

$\sqrt{2(x-1)} = \sqrt{(y+2)^2}$

$y + 2 = \sqrt{\pm 2(x-1)}$

$y = \pm\sqrt{2(x-1)} - 2$

$\mathbf{h(x) = \pm\sqrt{2(x-1)} - 2}$

NOTE: without the "$\pm$", graph would only be a (sideways) half-parabola

WR Question 2

(a) ❶ Vert. str. by factor of ¾, **then** vert. translation 6 units down

Next, vert shift 6 down….

$y=\frac{3}{4}(2x^2+4x+8)$ ➡ $y=\frac{6}{4}x^2+\frac{12}{4}x+\frac{24}{4}$ ➡ $\boldsymbol{y=\frac{3}{2}x^2+3x+6}$ ➡ $y=\frac{3}{2}x^2+3x+6-\mathbf{6}$ ➡ $\boldsymbol{g(x)=\frac{3}{2}x^2+3x}$

❷ *On $f(x)$ vertex is $(4,2)$, then on $g(x)$ its $(1,2)$* ➡ **horiz. shift 3 left**

$-$ sign is outside for **vertical reflection**, which would make the y-coord of the vertex -2. However, on $g(x)$ the y-coord is 2. So graph must also have been vertically translated 4 units up.

➡ $\boldsymbol{(x,y)\rightarrow(x-3,-y+4)}$

❸ From mapping rule $(x,y)\rightarrow(2x,y)$ we can see that a horiz str, factor of ½ was applied

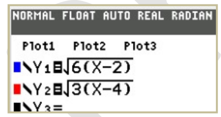
Check on your calc!

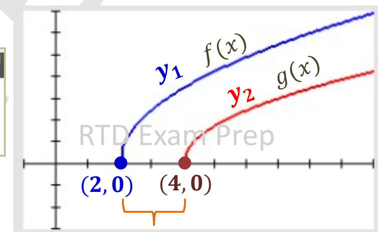
So, working backward from $g(x)$ to $f(x)$, we should apply a horiz str, **factor of 2**.

$f(x)=\sqrt{3(\mathbf{2x}-4)}$ *Replace $\boldsymbol{x}$ with $\mathbf{2x}$ in $g(x)$*

$f(x)=\sqrt{3*2(x-2)}$ *Factor out the **2** "inside"*

➡ $\boldsymbol{f(x)=\sqrt{6(x-2)}}$ *or, alternatively …..* $\boldsymbol{f(x)=\sqrt{6x-12}}$





Horiz. str, factor of 2 (works!)

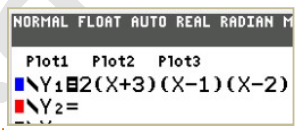
(b) RANGE of $f(x)$ is $\{\boldsymbol{y}\in\mathbb{R}\}$ *since $f(x)$ is a third degree (odd) polyomial function…. can also be determined by graphing*

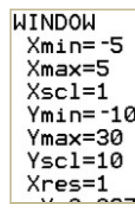
x-intercepts of $f(x)$ are $\boldsymbol{x=-3}$, $\boldsymbol{x=1}$, *and* $\boldsymbol{x=2}$

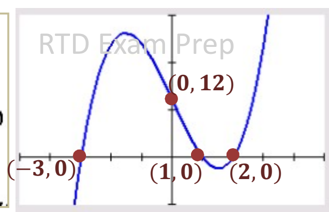
*Can be determined from **factors** …. or by graphing*

$f(x)=2(x+3)(x-1)(x-2)$

$x=-3$ $x=1$ $x=2$







This is all for $f(x)$, the original function!

y-intercept of $f(x)$ is $\boldsymbol{y=12}$

Set $x=0$ … $f(0)=2(\mathbf{0}+3)(\mathbf{0}-1)(\mathbf{0}-2)$

NOW, mapping rule here is $(x,y)\rightarrow(-x,3y)$ *(horiz refl, vert. str. of 3)*

So….x-intercepts of $g(x)$ are $\boldsymbol{x=-2}$, $\boldsymbol{x=-1}$, *and* $\boldsymbol{x=3}$ *Each becomes negative of original x-int.*

The y-intercept of $g(x)$ is $\boldsymbol{y=9}$ *Mult. y-int. of $f(x)$, which is 12, by 3/4*

The RANGE of $g(x)$ is $\{\boldsymbol{y}\in\mathbb{R}\}$ *No change to RANGE of $f(x)$, since it's all reals!*

NOTE: We need not find an equation for $g(x)$ (though you may to help justify your answers)