Additional WR Question 1:

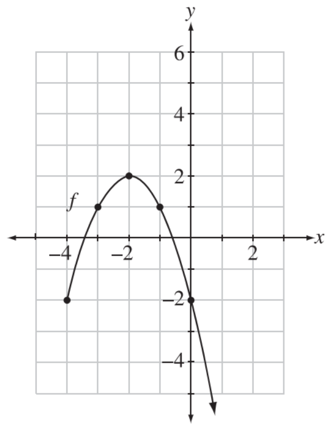
The graphs of two functions, $f(x)$ and $g(x)$, are shown on the right.

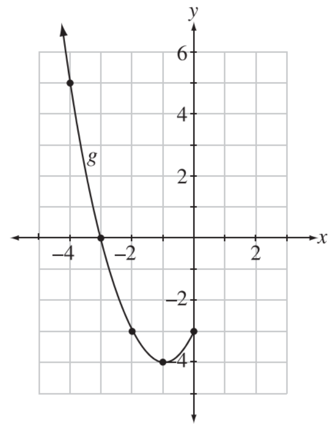
Three new functions, $j(x)$, $k(x)$, and $h(x)$, are defined by:

$$j(x) = (f \circ g)(x)$$

$$k(x) = \frac{f(x)}{g(x)}$$

and $h(x) = g(x) - f(x)$

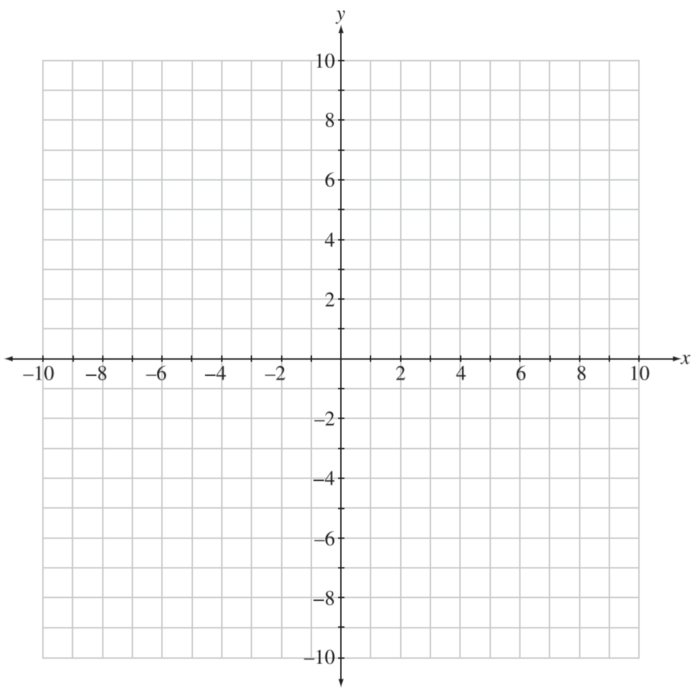




Written Response—5 marks

1. **a.** **Determine** which function, $j(x)$ or $k(x)$, has the larger value when $x = 0$. **[2 marks]**

b. **Sketch** the graph of $h(x)$ on the coordinate plane provided below. Identify the coordinates of the y-intercept, and state the domain of the function. **[3 marks]**



Coordinates of the y-intercept: ____________

Domain: ______________

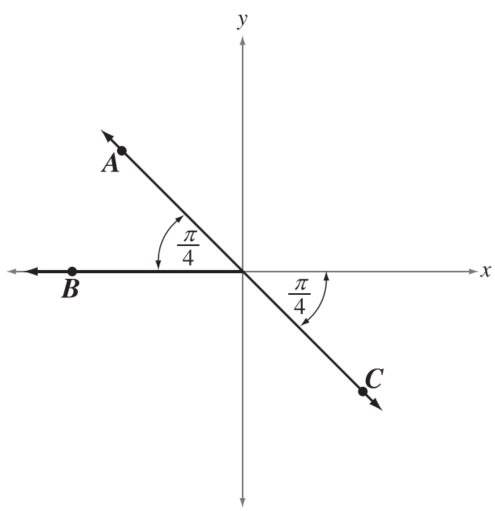
Additional WR Question 2:

Written Response—5 marks

2. **a.** **Algebraically solve** the equation $\sec^2\theta + \sec\theta = 2$, where $2\pi \le \theta \le 3\pi$. State the solution as **exact** values. **[3 marks]**

Use the following information to answer the next part of the written-response question.

Points A, B, and C are on the terminal arms of angles drawn in standard position, as shown below. These angles are the solutions for θ to a single trigonometric equation.



b. State the general solution for this equation. **[2 marks]**

Additional WR Question 3:

Use the following information to answer written-response question 3.

Students in a math class are creating and exchanging encoded messages with a partner.

Written Response—5 marks

3. **a.** Given that 630 different pairs of students are possible, **algebraically determine** the number of students in the math class. **[3 marks]**

Use the following information to answer the next part of the written-response question.

A student is asked to encode the word **FACTOR** by replacing each letter of the word with a different math symbol. There are 10 math symbols available. The student creates a key, a list of his letter-symbol replacements, and gives it to his partner to use to decode the word.

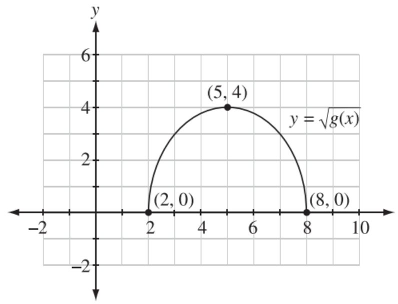
b. **Explain** how you would determine the number of different keys that can be created for the word **FACTOR**, and **determine** the number of possible keys using your strategy. **[2 marks]**

Additional WR Question 4:

Use the following information to answer part (a) of Question 4:

The graph of $h(x) = x^2$ is transformed into the graph of $y = g(x)$, whose equation can be written in the form $g(x) = a(x - h)^2 + k$.

The graph of $y = \sqrt{g(x)}$, shown on the right, has a maximum at the point (5, 4).

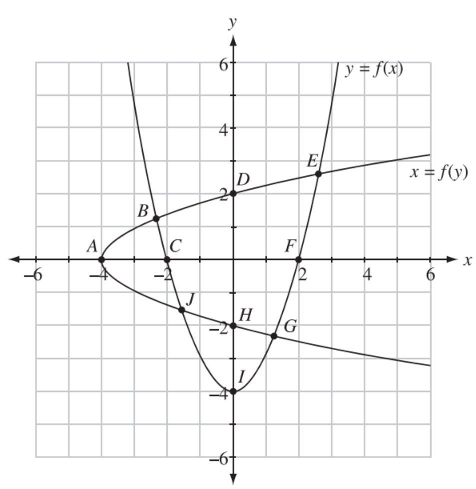


Written Response—5 Marks

a. **Sketch** the graph of $y = g(x)$ on the grid provided below, and clearly label the x-intercepts and maximum. **Describe** one possible sequence of transformations that could be applied to the graph of $y = h(x)$ to produce the graph of $y = g(x)$. **[3 marks]**

Use the following information to answer part (b) of Question 4:

The graph of the quadratic function $y = f(x)$ is transformed to create the graph of $x = f(y)$. The graphs of $y = f(x)$ and $x = f(y)$ and 10 labelled points are shown below.



b. Identify the invariant points and **explain** why they are invariant. **[2 marks]**

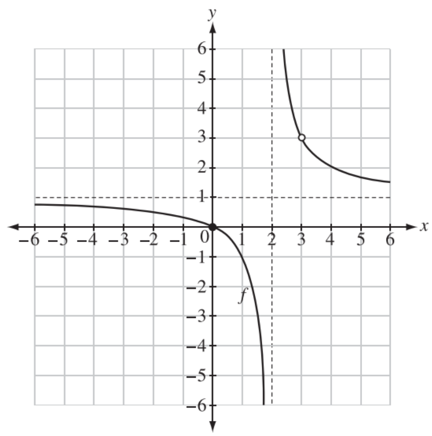
All questions are from the Alberta Education website

RTD LEARNING

Additional WR Question 5:

Use the following information to answer part (a) of Question 5:

The graph of $y = f(x)$ and the equation of $y = g(x)$ are shown on the right



$$g(x) = \frac{x+3}{x-2}$$

Written Response—5 marks

a. Compare the intercepts, equations of the asymptotes, and domains of $y = f(x)$ and $y = g(x)$. **[3 marks]**

Use the following information to answer part (b) of Question 5:

Betty is creating the function $h(x) = \frac{(x-a)(x-b)}{(x-c)}$ using the following values for a, b, and c:

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

Betty would like the graph of the function to have a vertical asymptote and two distinct x-intercepts, one of which is negative. She may use the provided values more than once.

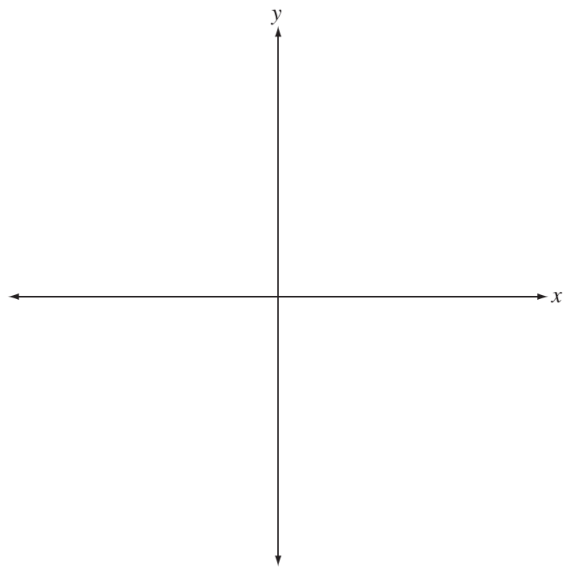
b. Provide an example of an equation Betty could use to represent the function, and **determine** the total number of different graphs that would meet Betty's criteria. **[2 marks]**

Additional WR Question 6:

Written Response—5 Marks

a. **Determine** the value of Angle θ in standard position such that $\csc\theta = -\sqrt{2}$, where $540° \leq \theta \leq 630°$. **Sketch** and label Angle θ in standard position on the Cartesian plane below. **[2 marks]**

$\theta =$ ___________



Use the following information to answer part (b) of Question 6:

> The terminal arm of Angle β intersects the unit circle at Point $P\left(m, -\frac{\sqrt{3}}{4}\right)$, where $\sec\beta > 0$.

b. **Algebraically determine** the exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$ in the form $\frac{\sqrt{a}+\sqrt{b}}{c}$, where $a, b, c \in N$. **[3 marks]**

SOLUTIONS *(possible solutions for each WR question)*

Question 1: Part (a)

$j(x) = (f \circ g)(x)$ $\qquad$ $k(x) = \dfrac{f(x)}{g(x)}$

$j(x) = \left(f\left(g(x)\right)\right)$ $\qquad$ $k(0) = \dfrac{f(0)}{g(0)}$

$j(0) = \left(f\left(g(0)\right)\right)$ $\qquad$ $k(0) = \dfrac{-2}{-3}$

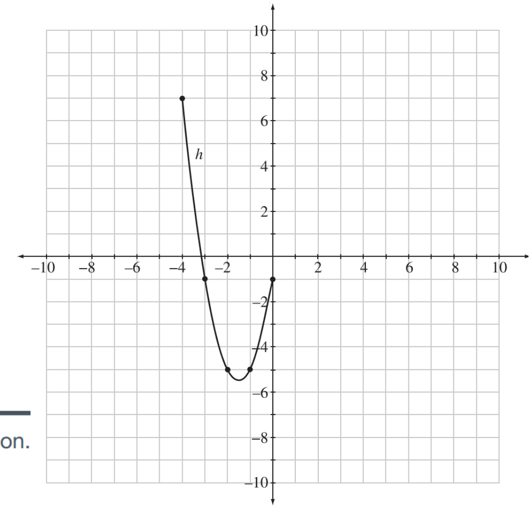
$j(0) = f(-3)$ $\qquad$ $k(0) = \dfrac{2}{3}$

$j(0) = 1$

The value of $j(x)$ is larger than the value of $k(x)$ when $x = 0$.

Part (b)

x	$g(x)$	$f(x)$	$g(x) - f(x)$	(x, y)
−4	5	−2	7	(−4, 7)
−3	0	1	− 1	(−3, −1)
−2	−3	2	− 5	(−2, −5)
−1	−4	1	− 5	(−1, −5)
0	−3	−2	− 1	(0, −1)



Coordinates of the y-intercept: $(0, -1)$

Domain: $D: \{x \mid -4 \le x \le 0,\ x \in R\}$ or $[-4, 0]$

Note: The domain can be written in either full set notation or interval notation. The y-intercept must be written as an ordered pair to receive full marks.

Question 2: Part (a)

$$\sec^2\theta + \sec\theta - 2 = 0$$
$$(\sec\theta + 2)(\sec\theta - 1) = 0$$

Therefore, $\sec\theta + 2 = 0$ or $\sec\theta - 1 = 0$

$\sec\theta = -2$ or $\sec\theta = 1$

$\cos\theta = -\dfrac{1}{2}$ or $\cos\theta = 1$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$ or $\theta = 0$

In the domain $2\pi \le \theta \le 3\pi$,

$\theta = \dfrac{2\pi}{3} + 2\pi$ and $\theta = 0 + 2\pi$

$\theta = \dfrac{8\pi}{3}$ and $\theta = 2\pi$

The solutions are $\theta = 2\pi$ and $\theta = \dfrac{8\pi}{3}$.

Part (b)

The three points are on the terminal arms of $\theta = \dfrac{3\pi}{4}, \pi$ and $\dfrac{7\pi}{4}$.

These angles represent the solutions to the equation in the domain $0 \le \theta \le 2\pi$.

As $\dfrac{3\pi}{4}$ and $\dfrac{7\pi}{4}$ have a difference of π, the general solution is:

$$\theta = \frac{3\pi}{4} + n\pi,\ \theta = \pi + 2n\pi,\ n \in I$$

Note: The general solution can be written as three separate statements.

Question 3: Part (a)

$${}_nC_2 = 630$$
$$\frac{n!}{(n-2)!2!} = 630$$
$$\frac{n(n-1)(n-2)!}{(n-2)!2!} = 630$$
$$\frac{n(n-1)}{2} = 630$$
$$n(n-1) = 1\,260$$
$$n^2 - n - 1\,260 = 0$$
$$(n-36)(n+35) = 0$$
$$n = 36 \text{ and } n = -35$$

It is not possible to have a negative number of students
So $n = -35$ is **extraneous**.
There are 36 students in the class

Part (b)

Possible Solution 1: Fundamental counting principle - there are 10 symbols available for the F, 9 symbols for the A, and so on…

$$\underset{\text{F}}{\underline{10}} \bullet \underset{\text{A}}{\underline{9}} \bullet \underset{\text{C}}{\underline{8}} \bullet \underset{\text{T}}{\underline{7}} \bullet \underset{\text{O}}{\underline{6}} \bullet \underset{\text{R}}{\underline{5}} = 151\,200$$

There are 151 200 different keys.

Possible Solution 2: Because the symbols will be arranged in a definite order, the problem can be solved using permutations.

$${}_{10}P_6 = 151\,200$$

Possible Solution 3: Because 6 symbols are being chosen from the set of 10, the first part can be solved using combinations. They must then be assigned to a specific letter in the word using the fundamental counting principle

$${}_{10}C_6 \times 6! = 151\,200$$

Question 4: Part (a)

The graph of $y = g(x)$ will have the same x-intercepts as the graph of $y = \sqrt{g(x)}$ but a maximum point at (5, 16).

The graph of $g(x)$ will also extend below the x-axis

A possible equation of $y = g(x)$:

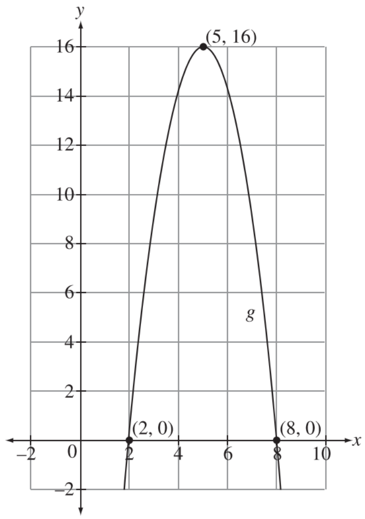
$$g(x) = a(x-5)^2 + 16$$

$$0 = a(8-5)^2 + 16$$

$$-16 = a(3)^2$$

$$a = -\frac{16}{9}$$

$$g(x) = -\frac{16}{9}(x-5)^2 + 16$$



A sequence of transformations that could be applied to $y = h(x)$ to result in $y = g(x)$ is

- a vertical stretch about the x-axis by a factor of $\frac{16}{9}$
- a reflection in the x-axis
- a horizontal translation 5 units right
- a vertical translation 16 units up

Part (b)

The invariant points are labelled on the graph as E and J.

First Possible Explanation: The graph of the inverse of a relation can be created by reflecting the graph of $y = f(x)$ in the line $y = x$. Any points on the graph of $y = f(x)$ that intersect with the reflection line $y = x$ would remain unchanged and therefore be invariant.

Second Possible Explanation: When a function is transformed into its inverse relation, the x-coord. of each point becomes the y-coord. of the corresponding point. (and vice-versa) This means that invariant points will only exist when the value of x is equal to the value of y.

Question 5: Part (a)

For $g(x)$: $g(x) = \frac{x+3}{x-2} \rightarrow g(x) = \frac{x-2+5}{x-2}$ → $g(x) = \frac{5}{x-2} + \frac{x-2}{x-2}$ → $g(x) = \frac{5}{x-2} + 1, \text{ where } x \neq 2$

The graph of $y = g(x)$, when compared to $y = \frac{1}{x}$, has been translated 1 unit up so the horizontal asymptote is $y = 1$. The vertical asymptote is $x = 2$, which means the domain is $\{x | x \neq 2, x \in R\}$.

x-intercept: $0 = \frac{x+3}{x-2}$

$0 = x + 3$ → $x = -3$

y-intercept: $y = \frac{0+3}{0-2}$

$y = -\frac{3}{2}$

The graphs of $y = f(x)$ and $y = g(x)$ have different x- and y-intercepts. The graph of $y = f(x)$ has an x-intercept at $(0, 0)$ and a y-intercept at $(0, 0)$. The graph of $y = g(x)$ has an x-intercept at $(-3, 0)$ and a y-intercept at $\left(0, -\frac{3}{2}\right)$.

REMEMBER: Intercepts **must** be expressed as ordered pairs!

Both graphs have a horizontal asymptote at $y = 1$ and a vertical asymptote at $x = 2$, but the graph of $y = f(x)$ also has a point of discontinuity at $x = 3$, meaning the domains will be different. The domain of $y = g(x)$ is $\{x \mid x \neq 2, x \in R\}$ and the domain of $y = f(x)$ is $\{x \mid x \neq 2, x \neq 3, x \in R\}$.

Part (b)

A possible example could be:

$$h(x) = \frac{(x+2)(x-5)}{(x-3)}.$$

To determine the total number of different graphs:

The value of a or b must be −3, −2, or −1 (**3 options**) while the value of the other parameter must then be 0, 1, 2, 3, 4, or 5. (**6 options**)

The value of c cannot be the same as the value of a or b; otherwise, the graph would have a point of discontinuity instead of a vertical asymptote. Therefore, there are **7** remaining unused options for the factor in the denominator.

Using the fundamental counting principle, 3 × 6 × 7 = **126** different graphs would meet Betty's criteria.

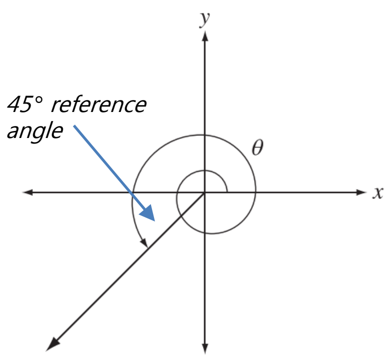
Question 6: Part (a)

$\csc\theta = -\sqrt{2}$

$\sin\theta = -\frac{1}{\sqrt{2}}$ } *NOTE: We recognize this as,* $-\frac{\sqrt{2}}{2}$ *which we know is associated with a 45° reference angle*

The terminal arm is in quadrant III or IV and the reference angle is $45°$.

The angle between $540° \le \theta \le 630°$ with a reference angle of $45°$ is $\theta = 585°$.



Part (b)

If Point $P\left(m, -\frac{\sqrt{3}}{4}\right)$ intersects the unit circle, then

$$m^2 + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1^2$$

$$m^2 = 1 - \frac{3}{16}$$

$$m^2 = \frac{13}{16}$$

$$m = \pm\frac{\sqrt{13}}{4}$$

Because $\sec\beta > 0$, Angle β is in quadrant IV and $m = \frac{\sqrt{13}}{4}$.

Therefore, Point P is located at $\left(\frac{\sqrt{13}}{4}, -\frac{\sqrt{3}}{4}\right)$ and $\cos\beta = \frac{\sqrt{13}}{4}$ and $\sin\beta = -\frac{\sqrt{3}}{4}$.

So, $\cos\left(\beta + \frac{\pi}{6}\right) = \cos\beta\cos\frac{\pi}{6} - \sin\beta\sin\frac{\pi}{6}$

$$= \left(\frac{\sqrt{13}}{4}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{4}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{39}}{8} + \frac{\sqrt{3}}{8}$$

$$\cos\left(\beta + \frac{\pi}{6}\right) = \frac{\sqrt{39} + \sqrt{3}}{8}$$

The exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$ is $\frac{\sqrt{39} + \sqrt{3}}{8}$.